The Operational Amplifier

By Tim Groth, January 12, 2015

The operational amplifier is used as a basic analog building block by Electrical Engineers to accomplish various functions such as amplification, filtering, adders, subtractors and various other versatile purposes. In many cases the Engineer can consider the op amp as ideal by means of mathematical simplifications, mostly when one number is at least ten times larger than the other and they are added together the smaller number can be completely ignored. If a basic model can be used for a circuit, and it is satisfactory, then a significant simplification occurs in mathematical equations, understanding the circuit functionality, and a significant time savings in design also occurs.

We use a simplified model of the op amp quite often so it is interesting to see the mathematics that lead to this. A key parameter of a real world op amp that leads to this is the high gain of the amplifier and the use of negative feedback to force the circuit to operate in a linear region. Some of the ideal characteristics of an op amp are

1. Infinite input impedance.
2. Zero output impedance
3. Infinite voltage gain
4. Infinite bandwidth.

The schematic symbol of the op amp is shown here in and it's a high gain differential amplifier.

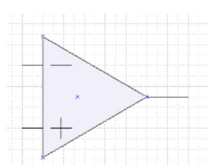

Figure 1 - The op amp

Applying negative feedback and also applying an input voltage source and showing it in a classic control loop results in Figure 2. The equations that describe this control loop are immediately below.

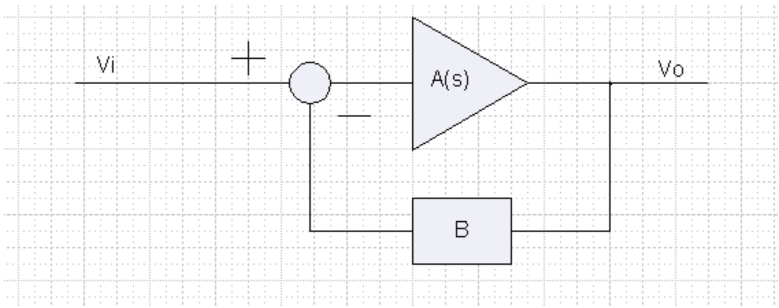

Figure 2 - Classic control loop with feedback and voltage source.

$$Vo = \left(Vi - VoBA(s)\right) A(s)$$

$$Vo + VoBA(s) = \; Vi\, A(s)$$

$$Vo\left(1 + BA(s)\right) = Vi\, A(s)$$

$$\frac{Vo}{Vi} = \frac{A(s)}{1+BA(s)}$$

Here's where the engineering comes in. In both an ideal op amp and a real op amp the Gain A(s) is much much larger than 1. Mathematically this is

$$BA(s) \gg 1$$

$$\frac{Vo}{Vi} = \frac{A(s)}{1+BA(s)}$$

So in the above formula the 1 in the denominator gets absorbed into the $BA(s)$ term which then allows A(s) to be cancelled in both the numerator and denominator. This leaves

$$\frac{Vo}{Vi} = \frac{1}{B}$$

Taking an example look at the following Figure and equations.

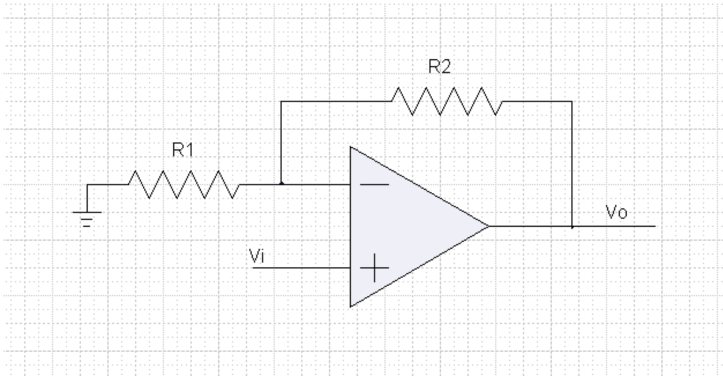

Figure 3 - Example Circuit

$$B = \frac{R1}{R1 + R2}$$

$$\frac{Vo}{Vi} = \frac{1}{B} = \frac{R1{+}R2}{R1} = 1 + \frac{R2}{R1}$$

Now consider the example circuit slightly redrawn in Figure 4. The input impedance r_i is calculated looking into the op amp from the Vi voltage source.

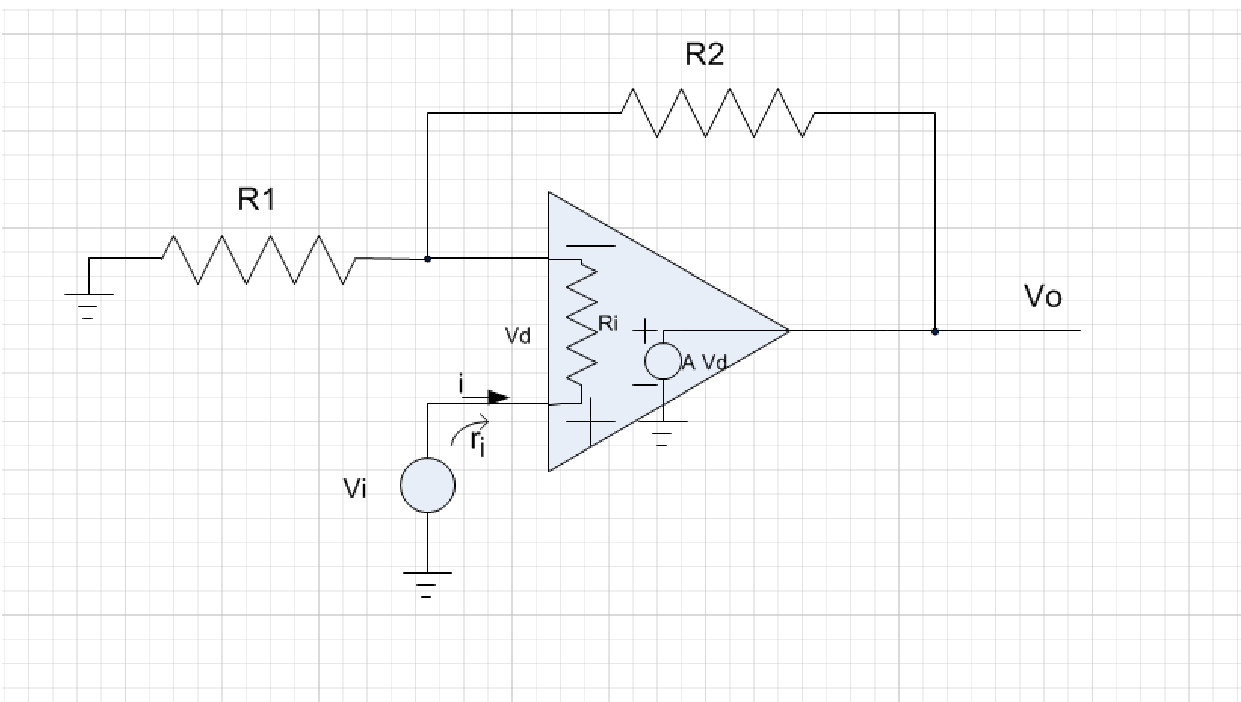

Figure 4 - Redrawn to show input impedance

$$i = \frac{Vd}{R_i} = \frac{Vo}{AR_i}$$

From above

$$Vo = Vi\,(1 + \frac{R2}{R1})$$

And substituting this in gives

$$i = Vi\frac{1+\frac{R2}{R1}}{ARi}$$

$$r_i = \frac{Vi}{i} = \frac{AR_i}{1 + R2/R1}$$

Again simplifying

$$r_i = AR_i$$

In a practical op amp R_i is about 1 M Ω, but looking into the op amp the impedance is multiplied by roughly the gain of the op amp as shown by the above equation. The current flowing into or out of either the plus or minus lead is very close to 0 even in a practical op amp.

Note that

$$Vd = \frac{Vo}{A} \approx 0$$

Vd approaches 0 in a practical op amp because A in the denominator is so large compared to Vo in the numerator. As already stated the current i is negligible to the current through R1 and R2. Thus when the positive lead is at ground, also there is essentially no current going into the – lead, and Vd is very close to 0. This is what is called a virtual ground as shown in Figure 5.

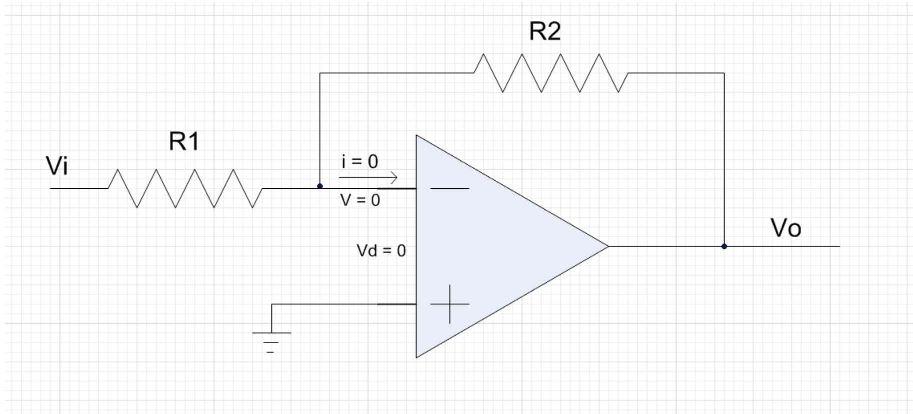

Figure 5 - i = 0, and v = 0 at the - lead

Ok so one more slight modification to the circuit to take a look at output impedance looking into the output of the op amp.

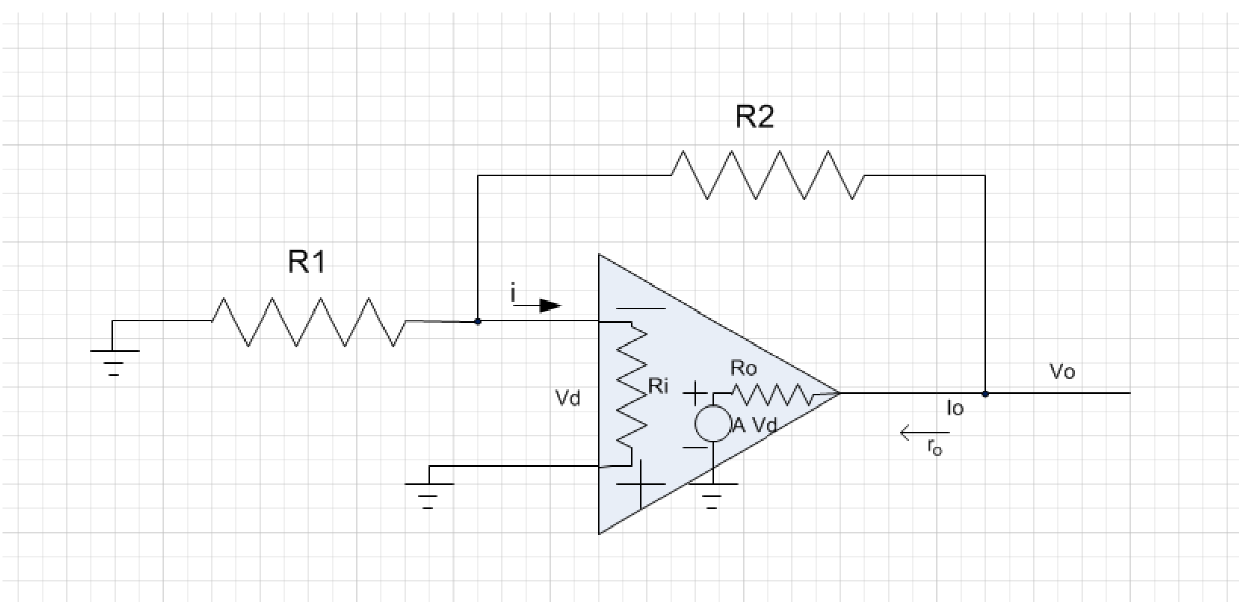

Figure 6 - Modified to show output impedance

The output impedance r_o is giving by

$$r_o = \frac{Vo}{Io}$$

$$I_o = \frac{Vo - AVd}{Ro}$$

$$-Vd = \frac{R1}{R1 + R2}\,Vo$$

So now substituting this equation into the one above it

$$Io = \frac{Vo + A\frac{R1}{R1 + R2}\;Vo}{Ro}$$

$$\frac{Vo}{Io} = \frac{Ro}{1 + \frac{AR1}{R1 + R2}}$$

Once again to simplify, A even in a practical op amp is so large the 1 in the denominator is absorbed and the resistor divider term times a large number is still a large number resulting in

$$ro = \frac{Ro}{A}$$

A is so large that this term is roughly 0. It's interesting that in all cases the very large gain of the amplifier was to an advantage.

A model with infinite input impedance and 0 output impedance and easily controlled with passive resistors or other components is very nice. The high impedance input does not disturb the output of the circuit that's driving it, and the output stage at a very low impedance can drive just about any load without the load its driving effect its operation. This model works very well at lower frequencies such as audio frequencies. When dealing with higher frequencies more detailed models can be used incorporating the gain bandwidth of the op amp and so on.

References:

1. *A Single-Supply Op-Amp Circuit Collection*, Texas Instruments Application Report, Literature Number SLOA049.
2. Paul R. Gray, Robert G. Meyer, “Analysis and Design of Analog Integrated Circuits”, 1977 1984.
3. K. Lal Kishore, “Operational Amplifiers and Linear Integrated Circuits”, 2007

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